

Theoretical and Educational Challenges with Enactivist Approaches to Mathematical Cognition

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> Abstract • Shvarts and Abrahamson bridge enactivism with Vygotsky's socio-cultural theory and Bernstein's coordination dynamics, and ground the proposed ideas with a case study. I question the interpretation of enactivism for mathematics educators, and the applicability of the proposed ideas to explain offline mathematics cognition.

« 1 » In their target article, Anna Shvarts and Dor Abrahamson interpret semiotic mediation of mathematical ideas as intercorporeal and sensorimotor coordination in a social and physical context. While they mainly argue for an enactivist epistemology to explain mathematical learning, they supplement enactivist theorizing with an amalgamation of Lev Vygotsky's (1978) cultural-historical view of artifacts' mediating role in acquiring higher-order skills and Nikolai Bernstein's (1967) movement and coordination dynamics. In what follows, I first interpret how the proposed approach can contribute to a wider understanding of embodied cognition (EC) in education, and, secondly, I question the applicability of the proposed ideas to mathematical cognition and learning in general, especially in the absence of a shared physical and semiotic context between two interlocutors.

Enactivism and education

« 2 » The authors argue that attempts to reformulate educational theories based on embodiment perspectives have not yet satisfactorily theorized cognition beyond human-environment sensorimotor coupling. The monistic approach proposed is intended to explain mathematical cognition as an extension of sensorimotor skills and resolve an ontological discontinuity between materiality and mental cognition. Combined with the authors' previous work (e.g., Abrahamson

2009; Shvarts et al. 2021), the proposed approach and the accompanying case study can be construed as a step toward developing an enactivist theory of mathematical thinking, learning, and teaching.

« 3 » Even though EC has been discussed widely in educational research, its impact on teacher training and practice has been limited. One reason for this is the theoretical gap between common notions about learning as mental activity alone and difficulties with making sense of the premises of the embodied views, particularly the radical and enactivist ones. Situated and bodily interactions with the environment supporting learning are not a foreign concept to teachers. For example, Jean Piaget's theories on the centrality of sensorimotor interactions in development, the use of math manipulatives, and constructivist learning design practices all share the common notion that bodily interactions have some form of relation with learning and cognitive development. It is perhaps due to this familiarity that embodied cognition is usually clumped with other approaches that assume a relationship between bodily activity and learning.

« 4 » EC is a mostly unfamiliar domain for teachers. Educational psychology textbooks and courses usually cover behaviorism, cognitivism, constructivism, and situated cognition as different paradigms of learning. Teachers' theorizing about learning involves a situation-specific and eclectic synthesis of these different paradigms. Even in learning design practices taking advantage of situated action in service of learning, bodily interactions are considered scaffolds for cognitive learning, helping shape "internal mental representations." Enactivism is a difficult approach to make sense of, both because it is orthogonal to common notions of learning as changes in mental structures and because its use to explain higher domains of cognition has been limited. My own experiences in teaching EC – to professional teachers in a master's level learning and cognition course over the last five years – have proved to me the challenges of breaking the rock-solid notions of learning and cognition as being "mental" and "representational."

« 5 » In the target article, the authors make enactivism and radical EC more accessible by representing a scenario where the learner makes sense of the mathemat-

cal content by exploiting the affordances of physical and cultural artifacts in the environment while coordinating the sensorimotor experience with a teacher. The environment is theorized as a system of *nested affordances* that enable both new and familiar forms of enactment. The learner can solve the trigonometric questions only by taking advantage of the artifacts and through corporeal coordination with an interlocutor. One question here is whether enactivism is only relevant when there are physical artifacts with affordances that facilitate the learning of conceptual content. In other terms, would it be possible to present an enactivist analysis of learning with a traditional method of learning trigonometry? Educators often get exposed to ideas of embodiment with learning-design examples that involve innovations with bodily interaction. This leads to the misconception that EC is a specific type of cognition that is only applicable when there is a direct relation between a designed corporeal experience and the semantic content. However, even when a student learns about the algebraic solution to $\sin a = \sin 2a$, principles of the enactivist theory proposed (e.g., affordances, action-perception loops, sensorimotor coordination) are equally applicable. It is even possible that teachers would have an easier time understanding that EC is not just a prescriptive learning-design theory, but an entirely different approach to learning and cognition, if it is presented with learning designs that are already familiar to them. So, to what extent should EC and embodied design (as a learning-design approach) be distinguished? And, can the proposed enactivist approach be applied to analyze learning in more traditional modes of learning that are taking place in classrooms? **Q1**

Mathematical cognition: Beyond enactivism?

« 6 » While theories of embodied cognition have high explanatory power in domains where there is a direct relation between the corporeal experience and the semantic content, there are "representation-hungry" (Clark & Toribio 1994) domains where cognition cannot be explained based on action-perception loops, dynamic assemblies of brain-body-environment systems, and affordances characterizing the organism's interactions with the environment. These

are domains where we, often unwillingly, resort to abstract mental representations, even if we espouse enactivist perspectives. Mathematics, being one of the most abstract domains of cognition, is one of them. This is mainly due to mathematical concepts not directly relating to daily, physical experiences and interactions. Early mathematical learning and development are relatively easy to explain from an enactivist perspective. For example, the development of number sense, and counting and arithmetic skills have been explained based on perceptuomotor abilities that allow object recognition, early tactile and motor experiences with fingers, and later finger-counting experiences (Berteletti & Booth 2016; Soylu, Lester & Newman 2018 for reviews). Theories on how finger-based interactions support development of mathematical abilities have embodied flavors, yet cannot be called fully enactivist. For instance, finger counting is used for addition and finger-counting experiences and habits seem to have an impact on later addition performance, even in adults. Yet, these have been explained as a sensorimotor simulation of finger-related sensorimotor circuitry during addition performance. In a sense, while early sensorimotor interactions are thought to support number development, accounts of adults' number processing performance still employ a cognitivist approach. Our reliance on concepts such as cognitive representations and mental structures equally applies to different domains of cognition (e.g., memory, language, visuospatial abilities), and has led to calls for alternative approaches, more in line with the EC principles (Anderson 2014).⁶

« 7 » What about offline math cognition? When an adult person does mental arithmetic – for example, multiplication of two two-digit numbers – there is no social or physical context and no perception–action loops. How can we interpret this situation from an enactivist standpoint? The mathematical experience during the complex multiplication operation is quite subjective. Some of the transformations can be explained relatively clearly (e.g., “first separate the tens and ones

in the first number”), however even these descriptions do not capture the first-person experience during the procedure. In one of the few accounts of first-person mathematical experiences, Jacques Hadamard (1945) asked mathematicians to describe how they experienced mathematical insight. These descriptions show that mathematical insight involves, sometimes quite colorful, mixtures of internalized corporeal experiences (e.g., seeing numbers as faces or in colors). How we can use first-person experiences in studying mathematical cognition is not clear. Can we use perception–action loops or nested affordances when describing offline, internalized mathematical experiences? Q2 In experimental studies, first-person experiences are usually reduced to strategy differences. For example, when answering 49×6 one can follow $50 \times 6 = 300 \rightarrow 300 - 6 = 294$, or $40 \times 6 = 240 \rightarrow 9 \times 6 = 54 \rightarrow 240 + 54 = 294$. Different problem-solving strategies are assumed to have processing differences (e.g., differential working-memory load; Tronsky 2005) and involve different neural resources. So far, the best that can be done with offline cognition is to reduce the first-person accounts to strategy differences and associate different information-processing models with each strategy (even if this means sur-rendering to cognitivism).

« 8 » Reducing first-person experiences to strategy differences leads to talking about them in a disembodied manner. The multiplication strategies in the example do not constitute experiences, but different computational steps. We do not have mature research methodologies that guide us in capturing offline first-person experiences in a way that we can use in experimental studies, even though there have been some attempts at that. *Neurophenomenology* (see Stuart, Pierce & Beaton 2013 for a special issue on this topic) is one such attempt, where the reported first-person experiences guide the analysis of experimental data. Yet, even though they are promising, such methods are not yet widely used.

« 9 » According to the enactivist approach to mathematical cognition that the authors espouse, mathematical abilities, like other domains of cognition, rely on perception–action loops, sensorimotor coupling with the environment, and affordances formed as a result of corporeal experienc-

es. However, we are at an impasse when it comes to explaining higher mathematical skills. *Higher* here refers to any mathematical skill that we cannot directly relate to interactions within a physical and social context. Further, even when we can explain the acquisition of a new math concept from an enactivist standpoint, as Shvarts and Abrahamson did with trigonometry in their case study, it is not clear how we can explain the use of previous learning in a new context and at a future time point. That is, what happens when the student in the case study is given the same trigonometry problem a year after the initial learning experience, but this time without the same physical artifacts? Is there a process where the situated nature of learning in the physical and social context is transformed to create abstract and generalized representations and skills? Or, does the learner rely on internalized perception–action loops and affordances? Internal simulation of previously acquired skills with physical artifacts is well exemplified with the mental-abacus phenomenon, where experts report imagining the abacus during calculation (Stigler 1984) and with neural evidence (Hanakawa et al. 2003) collaborating the simulation account. Should the simulation view for offline math cognition be supported and to what extent is the simulation account compatible with enactivism? Q3

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6| See also my preprint “A new cognitive ontology for numerical cognition,” retrieved on 8 June 2022 from <https://doi.org/10.31219/osf.io/3vx56>

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Concepts, Material Anchors and Interactivity: A Dialectic Perspective

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> Abstract • My present reflections will center on a point the authors present as an afterthought, but that seems pivotal: mathematical knowledge is not comprised of perception–action loops alone. Instead, “guided coordination of sensorimotor and semiotic activity” is held to be essential. Shvarts and Abrahamson do not elaborate on how this happens. My aim is to sketch what an account giving equal weight to semiotic and embodied facets might look like, and to clarify why paying attention to the details of their interplay is crucial for evaluating ontological claims such as the monist position defended by the authors. I will presently address four questions: (a) why failing to tackle the semiotic pole explicitly is a risky methodological choice, (b) what literature we can draw on to address the embodied–semiotic relationship, (c) what empirical criteria ontological claims might hinge on, and (d) why a dialectic (and non-dualist) approach offers a credible alternative to monism.

« 1 » With a study of mathematical learning, a cultural paragon of higher cognition, Anna Shvarts and Dor Abrahamson illustrate a new analytic apparatus based on a “hard case.” This explicit focus is to be applauded as it is essential to a well-grounded embodiment-oriented rebuttal of old cognitivist assumptions. A careful micro-analysis is precisely the way to address this, since zooming in on processuality can highlight factors cognitivists scarcely bother with. Readers can follow how complex effects emerge from smaller local, embodied activities and see how process-embedded perceptual and manipulation skills contribute to overall learning. In addition, a focus on embodied coordination (with cognitive artifacts and between teacher and pupil) supports new ways of thinking about pedagogy; learning is seen as an interactive process that

runs through recursive iterations of micro-alignment, rather than being the result of a teacher just “imparting content.” The analysis bears witness to a deep interwovenness and causal continuity of sensorimotor and thought processes. Showing how this unfolds over time is the target article’s central achievement.

Risks of skipping the “thought” layer

« 2 » Shvarts and Abrahamson’s (§46) concessionary afterthought concerning the semiotic pole implies that mathematical reasoning processes *can* accompany the observable embodied process, and contribute to its causality. We may speculate on the presence of such semiotic processes that are left implicit in the data: There is, for instance, a point of the process where the student is guided to a perceptual pattern, but cannot stabilize it yet or exploit it for a conceptual breakthrough. The reason could well be a missing conceptual process that later crystallizes and allows perceptual noticing.

« 3 » To evaluate what might be involved here, we may differentiate two types of semiotic/conceptual processes, those which reside “behind” the embodied, public process, and instances of micro-cognition that unfold “in the cracks” of the process. Reasoning processes in the cracks may escape notice easily, because they are extremely short-lived, partial solutions, or (perhaps tentative) ideas for directed further exploration. In contrast, reasoning processes “behind” the embodied process can be more systematic, of longer duration, and greater explicitness in terms of (partial) mental models that get refined or revised. Both types of semiotic processes seem likely candidates in the case of mathematics, and each needs to be accounted for.

« 4 » Leaving the semiotic processes implicit may also backfire on the theory. Putative hidden semiotic elements may lessen the import of the claim that insight hinges on the “spatial–temporal coordination between two bodies” (Figure 7 in the target article). Playing devil’s advocate, such coordination could be interpreted as a consequence of conceptual alignment processes that precede embodied alignments, rather than the other way around. After all, would two people sharing similar cognitive mod-